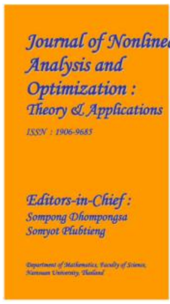
Journal of **N**onlinear **A**nalysis and **O**ptimization
Vol. 15, Issue. 1, No.1 : 2024
ISSN : **1906-9685**



PROPERTIES OF CENTRED POLYGONAL NUMBERS

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Abstract
Among the many relationships of numbers that have fascinated humans are those that suggest the arrangement of points representing numbers as particular geometrical figures. The Centred Polygonal Numbers are a less well known family of Figurate Numbers, this time generated by arranging points into a sequence of nested polygons of increasing size with a common centre. In this paper, we will explore some interesting mathematical properties of Centred Polygonal Numbers.

Keywords:Centred Polygonal Numbers, Centred Triangular Numbers, Centred Square Numbers, Centred Hexagonal Numbers.

1. Introduction

The centred polygonal numbers are class of figurate numbers, each formed by a central dot, surrounded by polygonal layers of dots with a constant number of sides.Such shapes were of importance not just in mathematics but also in other branches of science like crystallography and discrete geometry. Several mathematicians beginning the great Greek Era, two millennia ago, have considered such numbers and have established many numerical relationships pertaining to them. In this paper, we will be proving some basic results concerning Centred Polygonal Numbers.

2. Definition

The n^{th} centred polygonal number of order k is defined by

$$CP_{k,n} = \frac{kn(n+1)}{2} + 1 \qquad (1)$$

In particular if k =3,then

$$CP_{3,n} = \frac{3n(n+1)}{2} + 1 \qquad (2)$$

are called Centred Triangular Numbers.
Ifk=4, then

$$CP_{4,n} = \frac{4n(n+1)}{2} + 1 \qquad (3)$$

are called Centred Square Numbers
If k= 6, then

$$CP_{6,n} = \frac{6n(n+1)}{2} + 1 \qquad (4)$$

are called Centred Hexagonal Numbers.

3. Theorem 1

If $CP_{k,n}$are centred polygonal numbers, then theCentred Triangular Numbers are sum of three consecutive Triangular Numbers. That is, $T_{n-1} + T_n + T_{n+1} = CP_{3,n}$ (5)

Proof:

$$T_{n-1} + T_n + T_{n+1} = \frac{(n-1)(n-1+1)}{2} + \frac{n(n+1)}{2} + \frac{(n+1)(n+1+1)}{2}$$

$$= \frac{1}{2}[n^2 - n + n^2 + n + n^2 + 2n + n + 2] = \frac{1}{2}[3n^2 + 3n + 2] = \frac{3n^2 + 3n}{2} + 1$$

$$= \frac{3n(n+1)}{2} + 1 = CP_{3,n}$$

This completes the proof.

An illustration of expressing the centred triangular number 10 as sum of three triangular numbers is provided in Figure 1.

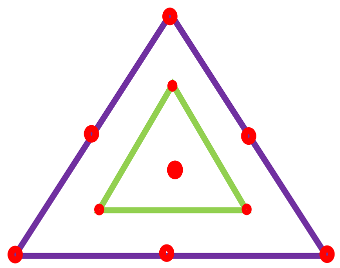

Figure 1: 1+3+6 =10

Centred Triangular Numbers are 1,4,10,19,31,46,85,109,136,166,199,235,274, …

4. Theorem 2

The Centred Square Numbers are sum of two consecutive Square Numbers.

$$CP_{4,n} = n^2 + (n+1)^2 \qquad (6)$$

Proof:

$$CP_{4,n} = \frac{4n(n+1)}{2} + 1$$

$$= 2n^2 + 2n + 1$$

$$= n^2 + (n+1)^2$$

This completes the proof.

5. Theorem 3

The Centred Hexagonal Numbers are difference of two consecutive cubes.

$$CP_{6,n} = (n+1)^3 - n^3$$

Proof: By Definition of Centred Hexagonal Numbers, we have

$$CP_{6,n} = \frac{6n(n+1)}{2} + 1$$

$$= 3n^2 + 3n + 1$$

$$= n^3 + 3n^2 + 3n + 1 - n^3 = (n+1)^3 - n^3$$

This completes the proof.

An illustration of expressing the centred hexagonal number 19 is shown in Figure 2.

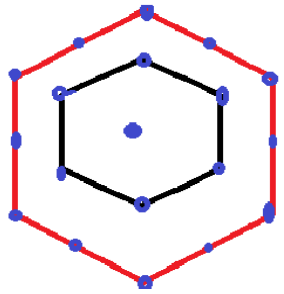

Figure 2

The Centred Hexagonal Numbers are 1, 7, 19, 37, 61, 91, 127, 169, 217, 271, 331, 397…

6.Theorem 4

The Centred Polygonal Numbers and Square Numbers satisfy the following equation

$$\frac{CP_{k,n}+CP_{k,n-1}-2}{k}=n^2$$

where $CP_{k,n}=\frac{kn(n+1)}{2}+1$

Proof: By definition, $CP_{k,n}=\frac{kn(n+1)}{2}+1$

$$\frac{CP_{k,n}+CP_{k,n-1}-2}{k}=\frac{\left(\frac{k}{2}n(n+1)+1\right)+\left(\frac{k}{2}n(n-1)+1\right)-2}{k}$$

$$=\frac{\frac{k}{2}\left(n(n+1)+(n-1)(n)\right)}{k}=2\,\frac{n^2}{2}=n^2$$

Hence, $\frac{CP_{k,n}+CP_{k,n-1}-2}{k}=n^2$

This completes the proof.

Conclusion

After defining Centred Polygonal Number of order k, we have derived four elementary properties related to such numbers. In particular, in theorem 1, we have proved that the Centred Triangular Numbers are sum of three consecutive Triangular Numbers. In theorem 2, we have proved that the Centred Square Numbers are sum of two consecutive square numbers. In theorem 3, we have proved that the Centred Hexagonal Numbers are difference of two consecutive cubes and finally in theorem 4, we have established a nice relationship between Centred Polygonal Numbers and Square Numbers. These elementary results will provide more insights upon understanding the patterns of Centred Polygonal Numbers. There are ample scope for proving more results and relationships as that of done in this paper.

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